**SI231b – Matrix Computations**
**Spring 2020-21, ShanghaiTech**

**Basic Information:**
Instructor: Prof. Ziping Zhao (https://zipingzhao.github.io)
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SI231b – Matrix Computations [4 credits: 3+1]
Website: http://si231.sist.shanghaitech.edu.cn
Lecture Time: Mon/Wed 3:00pm-4:40pm
Lecture Venue: Rm. 1D-106, SIST Building ~~Rm. 203, Teaching Center~~

**Description:**
Matrix analysis and computations are widely used in engineering fields — such as statistics, optimization, machine learning, computer vision, systems and control, signal and image processing, communications and networks, smart grid, and many more — and are considered key fundamental tools.
SI231b: Matrix Computations covers topics at an advanced or research level especially for people working in the general areas of Data Analysis, Signal Processing, and Machine Learning.
This course consists of several parts.

- The first part focuses on various matrix factorizations, such as eigendecomposition, singular value decomposition, Schur decomposition, QZ decomposition and nonnegative factorization.
- The second part considers important matrix operations and solutions such as matrix inversion lemmas, linear system of equations, least squares, subspace projections, Kronecker product, Hadamard product and the vectorization operator. Sensitivity and computational aspects are also studied.
- The third part explores presently frontier or further advanced topics, such as matrix calculus and its various applications, deep learning, tensor decomposition, and compressive sensing (or managing undetermined systems of equations via sparsity). Especially, matrix concepts are key for understanding and creating machine learning algorithms, and hence, a special focus will be given on how matrix computations are applied to neural networks.

In each part, the relevance to engineering fields is emphasized and applications are showcased.

**Textbooks:**

- Gene H. Golub and Charles F. Van Loan, Matrix Computations (Fourth edition), The John Hopkins University Press, 2013.
- Roger. A. Horn and Charles. R. Johnson, Matrix Analysis (Second Edition), Cambridge University Press, 2012.
- Jan R. Magnus and Heinz Neudecker, Matrix Differential Calculus with Applications in Statistics and Econometrics (Third Edition), John Wiley and Sons, New York, 2007.
- Gilbert Strang, Linear Algebra and Learning from Data, Wellesley-Cambridge Press, 2019.
- Carl D. Meyer, Matrix Analysis and Applied Linear Algebra, SIAM (Society for Industrial and Applied Mathematics), 2000.
- Alan J. Laub, Matrix Analysis for Scientists & Engineers, SIAM (Society for Industrial and Applied Mathematics), 2004.

**Prerequisite:**
Students are expected to have a solid background in linear algebra and know basic machine learning and signal processing. They are also expected to have research experience in their particular area and be capable of reading and dissecting scientific papers.

**Grading:**
Assignments: 30% (auditors too)
Mid-term exam: 40% (auditors too)
Final Project: 30% (homeworks and midterm are required to be passed)

**Course Schedule:**

| Date | Lec. | Topic | HW out | HW in |
|---|---|---|---|---|
| Feb-22 | 1 | Topic 0: Overview | | |
| Feb-24 | 2 | Topic 1: Basic Linear Algebra – I | | |
| Mar-1 | 3 | Topic 1: Basic Linear Algebra – II | | |
| Mar-3 | 4 | Topic 1: Basic Linear Algebra – III | HW1 | |
| Mar-8 | 5 | Topic 2: Linear Systems – I | | |
| Mar-10 | 6 | Topic 2: Linear Systems – II | | HW1 |
| Mar-15 | 7 | Topic 2: Linear Systems – III | | |
| Mar-17 | 8 | Guest Lecture: Solving Quadratic Systems (Phase Retrieval) | HW2 | |
| Mar-22 | 9 | Topic 3: Least Squares – I | | |
| Mar-24 | 10 | Topic 3: Least Squares – II | | HW2 |
| Mar-29 | 11 | Topic 3: Least Squares – III | | |
| Mar-31 | 12 | Topic 4: Orthogonalization and QR Decomposition – I | | |
| ~~Apr-5~~ | ~~13~~ | Qingming Festival | | |
| Apr-7 | 14 | Topic 4: Orthogonalization and QR Decomposition – II | HW3 | |
| Apr-12 | 15 | Topic 5: Eigenvalues, Eigenvectors, and Eigendecomposition – I | | |
| Apr-14 | 16 | Topic 5: Eigenvalues, Eigenvectors, and Eigendecomposition – II | | HW3 |
| Apr-19 | 17 | Topic 5: Eigenvalues, Eigenvectors, and Eigendecomposition – III | | |
| Apr-21 | 18 | Topic 6: Positive Semidefinite Matrices | HW4 | |
| Apr-26 | 19 | Topic 7: Singular Value Decomposition – I | | |
| Apr-28 | 20 | Topic 7: Singular Value Decomposition – II | | HW4 |
| May-3 | 21 | Topic 8: Least Squares Revisited – I | | |
| May-5 | 22 | Topic 8: Least Squares Revisited – II | HW5 | |
| May-10 | 23 | Topic 9: Kronecker Product and Hadamard Product | | |
| May-12 | 24 | Topic 10: Review | | HW5 |
| | **Course Project** | | | |
| Jun-7 | **Final Course Project Presentation** | | | |